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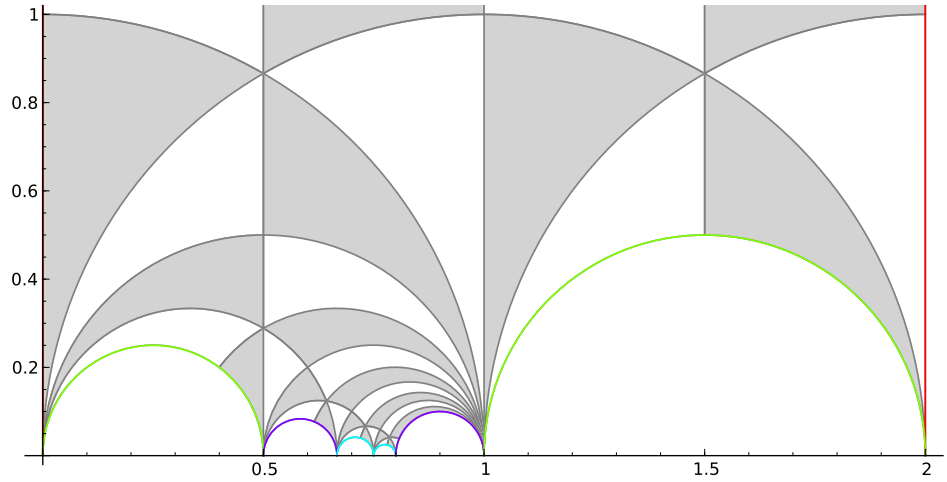
MODULAR FORMS - A COMPUTATIONAL APPROACH



A fundamental domain of a finite index subgroup of $\mathrm{SL}_2(\mathbb{Z})$

Modular forms play an important role in number theory, algebraic geometry and physics. Using new tools from an open source project called SAGE[SAG] it possible to access a wealth of information about the module space. This class will teach you how to calculate Eisenstein series, cusp forms and hauptmoduls and will give you some intuition about the algebraic structure of the modular group. The seminar will be based on the book Modular forms a computational approach by William Stein[Ste07] which is also available at `http://modular.math.washington.edu/books/modform/`. So bring your laptop and see what SAGE can do for you.

Prerequisites

You should be interested in taking an computational approach to a abstract problem.

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Time and place

The seminar will be held every

Tuesdays at 14hct
in seminar room I in the physics institute.

The pre-meeting will be

on Thursday, February the 6th at 11hct
in conference room I in Physics Institute

In this meeting we will give a short overview over the seminar and distribute the talks. If you are not able to come, write us a mail.

How this seminar works

Every student will give a talk. One week before the talk, we would like to have a short discussion with you to see if there are any problems.

References

[Apo90] T. M. Apostol. *Modular Functions and Dirichlet Series in Number Theory*. Graduate Texts in Mathematics. Springer-Verlag, 1990.

[Bru08] J.H. Bruinier. *The 1-2-3 of modular forms: lectures at a summer school in Nordfjordeid, Norway*. Universitext (1979). Springer, 2008.

[DS05] F. Diamond and J. Shurman. *A First Course in Modular Forms*. Graduate Texts in Mathematics 228. Springer, 2005.

[KL07] Chris A. Kurth and Ling Long. Computations with finite index subgroups of $psl_2(F)$ using farey symbols. October 2007.

[Kul91] R. S. Kulkarni. An arithmetic geometric method in the study of the subgroups of the modular group. *American Journal of Mathematics*, 113(6):1053–1133, 1991.

[SAG] Sage. `http://www.sagemath.org/`. [Acccessed: 10th November 2012].

[Ste07] W. Stein. *Modular forms, a computational approach*. Graduate studies in mathematics. American Mathematical Society, 2007.

Talks

0. Talk. **Installation of SAGE.** Stefan Krämer
In this first session everybody should bring their laptop and we help you to get SAGE running.

1. Talk. **General Introduction and a first look at SAGE.** NN
Definition of the modular group $\mathrm{SL}_2(\mathbb{Z})$and its basic properties: Generators, operation on the upper half plane, the fundamental domain and the Dedekind tessellation. The $\mathrm{SL}_2(\mathbb{Z})$-word problem. Definition of a modular form (of level one), its Fourier expansion. Cusp-Forms. Short introduction to SAGE. How is SAGE used to get modular forms? [Ste07, Chapter 1], [DS05, Chaper 1.1] and [Apo90]

The full modular group

2. Talk. **Hecke operators.** NN
[Ste07, Chapter 2.4]: Definition of a Hecke operator and basic properties, especially Proposition 2.29. Computing Hecke operators [Ste07, Chapter 2.5]

3. Talk. **Modular symbols.** NN
Modular symbols [Ste07, 3.2]. Manins trick, Manin symbols [Ste07, 3.3]. Hecke operators on Modular symbols [Ste07, 3.4]

4. Talk. **Character theory - optional.** NN
Chapter 4 of [Ste07]

Arithmetic subgroups of the full modular group

5. Talk. **Introduction to arithmetic subgroups.** NN
Subgroup of $\mathrm{SL}_2(\mathbb{Z})$of finite index. Quotient on the upper half plane as Riemann surface, fundamental domain, pairings, cusps. Definition of genus, elliptic points etc. How is this done in SAGE? Representation as permutation. Examples: congruence subgroups.

6. Talk. **Geometric aspects and dimension formulas.** NN
Generalize the definition of modular forms and functions. Dimension of the space of modular forms, Eisenstein series and cusp forms. How to find calculate forms in SAGE?

7. Talk. **Farey-Symbols.** NN
[KL07]: Definition of a special polygon, bijection with arithmetic subgroups, Farey-Sequence, Farey-Symbol. Algorithms: Construction, Group Membership, cosets. The original article [Kul91] is also nice to read.

8. Talk. **General modular symbols.** NN
Recall the definition of a modular symbol and generalize to arbitrary level, [Ste07, 8.1]. Manin symbols [Ste07, 8.2].

9. Talk. **Hecke operators.** NN
[Ste07, 8.3], general definition of Hecke operators, Hecke operators on Manin symbols. Perhaps Cuspidal Manin Symbols [Ste07, 8.4]. Hecke operators as self adjoined operator of the pairing, [Ste07, 8.5].

10. Talk. **Explicit examples.** NN
Begin with [Ste07, 8.7].
Calculate some examples as in [Ste07, 8.8]. Show how modular forms are calculated in sage, [SAG, `http://www.sagemath.org/doc/reference/modfrm/index.html`]

Applications and advanced topics

These talks will only be distributed if are enought students. In most cases you will need special literature, which we will give you.

11. Talk. **Newforms.**

12. Talk. **Elliptic curves.**

13. Talk. **Hejhals algorithm.**

14. Talk. **Dessins d'Enfants.**

15. Talk. **Periods.**

16. Talk. **Modular curves.**

17. Talk. **Moonshine-Theory.**