# Exercise-sheet 9 (July 11, 2017)

## 1 Homework - due Date: July 18, 2017 (25 points).

### 1.1 Landau free energy density and 1st order transitions (10pts)

Consider the Landau free energy density

$$\mathcal{L} = \alpha t\, \eta^2 + \frac{1}{2}\beta\, \eta^4 + \gamma\, \eta^3, \tag{1}$$

with $\alpha$ and $\beta$ positive, and the reduced temperature $t = 1 - T/T_c$.

Find the equilibrium solution $\eta \in \mathbb{R}$, and argue the presence of the cubic term allows for a first order phase transition.

Comment on the validity of your results.

### 1.2 Ginzburg-Landau theory for superfluids: differential equation for $\psi$ (10pts)

Given the Ginzburg-Landau U(1) free energy functional

$$L[\psi] = \int d^3r \left[\alpha\, |\psi|^2 + \frac{\beta}{2}\, |\psi|^4 + \frac{\hbar^2}{2m^*}\, |\nabla\psi|^2\right], \tag{2}$$

consider the displaced order parameter

$$\psi(\boldsymbol{r}) \to \psi(\boldsymbol{r}) + \epsilon(\boldsymbol{r}), \tag{3}$$

where $\epsilon(\boldsymbol{r})$ is small, and use the extremum principle $\delta L = 0$ to show that the equilibrium solution $\psi(\boldsymbol{r})$ satisfies

$$-\frac{\hbar^2}{2m^*}\nabla^2\psi + \alpha\psi + \beta|\psi|^2\psi = 0. \tag{4}$$

### 1.3 On phase rigidity and superflow (5pts)

In a condensate, the quantum equation of motion for the phase of the order parameter $\psi$ can be expressed in Hamiltonian dynamics

$$\hbar\frac{d\phi}{dt} = i[\phi, H] = -\frac{\partial H}{\partial N} \tag{5}$$

(a) Use the equation above to show that in a superfluid at chemical potential $\mu$, the equilibrium order parameter will precess with time according to

$$\psi(x,t) = \psi(x,0)\, e^{-i\mu t/\hbar}. \tag{6}$$

(b) If two superfluids with the same superfluid density, but at different chemical potentials $\mu_1$ and $\mu_2$ are connected by a tube of length $L$, show the superfluid velocity $v_s$ from $1 \to 2$ will accelerate according to the equation

$$\frac{d\,v_s}{dt} = -\frac{\hbar}{m}\frac{\mu_2 - \mu_1}{L}. \tag{7}$$